Chapter 5

The Comparison among ARMA-GARCH, -EGARCH, -GJR, and -PGARCH Models on Thailand Volatility Index

ABSTRACT

With the formulae of Volatility Index (VIX) which was launched by the Chicago Board Options Exchange (CBOE) in 2003, SET50 Index options is applied as a Thailand Volatility Index (TVIX). We estimate ARMA-GARCH, -EGARCH, -GJR and -PGARCH models for Thailand Volatility Index (TVIX). These models are the extension of ARCH process with various features to explain the obvious characteristics of financial time series such as asymmetric and leverage effect. As we apply TVIX with these models, the comparison and forecast are performed.

Keywords: Volatility index; Model selection; GARCH; Asymmetry effect; Time series.

5.1 Introduction

In recent years, financial crises impact global economy. The crises dramatically cause recession in commodities and money markets because of the liquidity shrinking. While the decrease in most assets occurs, an important figure in financial market, called volatility index, inversely turn. The price with high volatility reflects higher risk in holding such asset. The volatility can be calculated in a numerical value as an index known as Volatility Index.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index, VIX, which quickly became the benchmark for stock market volatility. In 2003, the CBOE made two key enhancements to the VIX methodology. The new VIX is based on an up-to-the-minute market estimation of expected volatility that is calculated by using real-time S&P500 Index (SPX) option bid/ask quotes. Until 2006, VIX was trading on the CBOE. The VIX options contract is the first product on market volatility to be listed on an SEC-regulated securities exchange. This new product can be traded from an options-approved securities account. Many investors consider the VIX to be the world's premier barometer of investor sentiment and market volatility, and VIX options are a very powerful risk management tool.

The early generation of GARCH models, such as the ARCH and GARCH models have the ability of reproducing another very important stylized fact, which is volatility clustering; that is, big shocks are followed by big shocks. However, only the magnitude of the shock, but not the sign, affects conditional volatility. Therefore, the first generation of GARCH models cannot capture the stylized fact that bad (good) news increase (decrease) volatility. This limitation has been overcome by the introduction of more flexible volatility specifications which allow positive and negative shocks to have a different impact on volatility. This more recent class of GARCH models includes the Exponential GARCH (EGARCH), the Glosten, Jagannathan, and Runkle-GARCH (GJR-GARCH) and the Power GARCH (PGARCH) model. Finally, a new class of GARCH models which jointly capture leverage effects and contemporaneous asymmetry, as well as time varying skewness and kurtosis, has been recently introduced by El Babsiri and Zakoian (2001). In a recent paper, Patton (2004) also analyzes the use of asymmetric dependence among stocks; that is, the fact that stocks are more highly correlated during market downturns.

In this paper, we applied VIX and compare the conditional variance among various GARCH models which are GARCH, EGARCH, GJR-GARCH and PGARCH models. Nevertheless, it should be pointed out that several empirical studies have already examined the impact of asymmetries on the forecast performance of GARCH models. The recent survey by Poon and Granger (2003) provides, among other things, an interesting and extensive synopsis of them. Indeed, different conclusions have been drawn from these studies. In fact, some studies find evidence in favor of asymmetric models, such as EGARCH, for the case of exchange rates and stock returns predictions. Examples include Cao and Tsay (1992), Heynen and Kat (1994), Lee (1991), and Pagan and Schwert (1990). Other studies find evidence in favor of the GJR-GARCH model. The studies of Taylor (2001) also examine the case of stock returns volatility, and Bali (2000) for interest rate volatility. For PGARCH, interesting evident can be found from the study of Sebastien Laurent which derives analytical expressions for the score of the PGARCH model of Ding, Granger, and Engle (1993).

The rest of the paper is organized as follows: Section 2 presents the CBOE VIX formula which the adaptation of the VIX to Thailand SET50 Index options, the Thailand Volatility Index (TVIX), can be estimated. Section 3 formally defines theory and process of GARCH, EGARCH, GJR-GARCH, and PGARCH models. The data is shown in section 4 which daily returns of TVIX are described. The estimation of ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models are shown in the final section. This section provides tables and figures of family of GARCH on Returns of TVIX and the comparison of test statistics, together with a brief conclusion.

5.2 Volatility Index

Estimating implied volatility from options is no straightforward method to extract the information. Whaley (2000) considered implied volatility as a fear gauge because option prices calculate implied volatility that represents a market-based estimate of future price volatility). Implied volatilities are the information by investors, financial news services and other finance professionals. The information content and forecast quality of implied volatility is an important topic in financial markets research.

Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981) and Jorion (1995) provided early assessments of the forecast quality of implied volatility and concluded that implied volatility outperforms historical standard deviations and is a good predictor of future volatility, although it might be biased. Christensen and Prabhala (1998) also found that implied volatility forecasts are biased, but dominate historical volatility in terms of ex ante forecasting power. Fleming (1998) used a historical volatility measure to show that implied volatilities outperform historical information.

Dennis *et al.* (2006) found that daily innovations in VIX contain very reliable incremental information about the future volatility of the S&P100 index. Other studies that attempt to forecast implied volatility or use the information contained in implied volatility to trade in option markets include Harvey and Whaley (1992), Noh *et al.* (1994), and Poon and Pope (2000).

The New VIX is more robust because it pools the information from option prices over the whole volatility skew, and not just from at-the-money options. The formula used in the new VIX calculation is given by the CBOE as follows:

$$\sigma^2 = \frac{2}{T}\sum_{t}\frac{\Delta K_i}{K_i^2}e^{RT}Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0}-1\right]^2, \tag{5.1}$$

where

σ = VIX / 100 (so that VIX = σ x 100),

T = Time to expiration (in minutes),

F = Forward index level, derived from index option prices (based on at-the-money option prices, the difference between call and put prices is smallest).

The formula used to calculate the forward index level is:

F = Strike price (at-the-money) + e^{RT} x (Call price – Put price),

where

R = risk-free interest rate is assumed to be 3.01% (for simplicity, the government T-bills 3 month contract interest rate is used, as the Thailand options contract is a 3 months contract);

T = $\{M_{\text{current day}} + M_{\text{settlement day}} + M_{\text{other days}}\}$/minutes in a year,

where

$M_{\text{current day}}$ = # of minutes remaining until midnight of the current day,

$M_{\text{settlement day}}$ = # of minutes from midnight until 9:45 am on the TFEX settlement day,

$M_{\text{other days}}$ = Total # of minutes in the days between the current day and the settlement day;

K_i = Strike price of i^{th} out-of-the-money option; a call if $K_i > F$ and put if $K_i < F$;

ΔK_i = Interval between strike prices - half the distance between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$.

K_0 = First strike below the forward index level, F.

$Q(K_i)$ = The midpoint of the bid-ask spread for each option with strike K_i.

(Note: ΔK_i for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK_i for the highest strike is the difference between the highest strike and the next lower strike.)

With the adaptation of the VIX calculation to Thailand SET50 index options, the Thailand Volatility Index (TVIX) can be estimated.

5.3. Theory

5.3.1 GARCH Model

GARCH model by Bollerslev (1986) imposes important limitations, not to capture a positive or negative sign of u_t, which both positive and negative shocks has the same impact on the conditional variance, h_t, as follows,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{5.2}$$

where $\omega > 0$, $\alpha_i \geq 0$ for i = 1,…,p and $\beta_j \geq 0$ for j = 1,…,q are sufficient to ensure that the conditional variance, σ_t, is non-negative. For the GARCH process to be defined, it is required that $\omega > 0$. Also, a univariate GARCH(1,1) model is known as ARCH(∞) model (Engle, 1982) as an infinite expansion in u_{t-i}^2. The α represents the ARCH effect and β represents the GARCH effect.

5.3.2 Exponential GARCH (EGARCH) Model

Exponential GARCH (EGARCH) model by Nelson (1991) is the logarithm of conditional volatility in order to capture asymmetries between positive and negative shocks that the leverage effect is exponential, as follows,

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left|\eta_{t-i}\right| + \sum_{j=1}^{q} \beta_j \log\left(\sigma_{t-j}^2\right) + \sum_{k=1}^{r} \gamma_k \left|\eta_{t-k}\right| \tag{5.3}$$

where $\eta_{t-i} = \frac{u_{t-i}}{\sigma_{t-i}}$ which η_{t-i} and $|\eta_{t-i}|$ capture the sign and size effects of the standardized shocks. There are no restrictions on the parameters in the model. The moment conditions of the model are also straightforward because the standardized shocks have finite moments. There is an leverage effect when $\gamma < \alpha < -\gamma$. This implies that the negative shocks increase volatility and vice versa.

5.3.3 Glosten, Jagannathan and Runkle (GJR-GARCH) Model

Glosten, Jagannathan and Runkle (GJR-GARCH) model by Glosten *et al.* (1993) is to capture possible asymmetric impacts of positive and negative shocks on the conditional variance for σ_t, as follows,

$$\sigma_t = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I(u_{t-i})) u_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}, \qquad (5.4)$$

where $I(u_{t-j})$ is an indicator function that equals to 1 if $u_{t-j} < 0$ and 0 otherwise. If p = q = 1, $\omega > 0, \alpha_1 \geq 0, \alpha_1 + \gamma_1 \geq 0$ and $\beta_1 \geq 0$ are sufficient conditions to ensure that the conditional variance h_t is non-negative. $\alpha_1(\alpha_1 + \gamma_1)$ gives the short-run persistence of positive (negative) shocks. If $\gamma_1 \neq 0$, the news impact is asymmetry. If $\gamma > 0$, there is a leverage effect that bad news increases volatility.

Lee and Hansen (1994) derived the log-moment condition for GARCH(1,1) of conditional volatility which is sufficient for the statistical properties of the QMLE to be consistent, as follows,

$$E(\log(\alpha_1 \eta_t^2 + \beta_1)) < 0 . \tag{5.5}$$

It is essential to note that the log-moment condition is a weaker regularity condition than the second moment condition. Therefore, the second moment is sufficient condition for consistency and asymptotic normality of the QMLE, as follows,

$$\alpha_1 + \beta_1 < 1 . \tag{5.6}$$

Moreover, McAleer *et al.* (2007) established the log-moment and second moment condition for GJR(1,1) as follows,

$$E(\log((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1)) < 0 \tag{5.7}$$

and

$$\alpha + (\gamma / 2) + \beta < 1 \tag{5.8}$$

Both moments are the sufficient conditions for the consistency and asymptotic normality of the QMLE for the GJR(1,1).

5.3.4 Power GARCH (PGARCH) Model

Power GARCH (PGARCH) model by Taylor (1986) and Schwert (1989) use the conditional standard deviation as a measure of volatility instead of the conditional variance. This model is generalized by Ding *et al.* (1993) using the PGARCH model as follows:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i \left(|u_{t-i}| - \gamma_i u_{t-i} \right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{5.9}$$

where $for\, i = 1,2,...,r\; and\, \gamma_i = 0\, for\, i > r, and\, r \le p.$

In the PGARCH model, if $\gamma \neq 0$, this captures asymmetric effects. The PGARCH model reduces to the GARCH model when $\delta = 2$ and $\gamma_i = 0$ for all *i*.

5.4 Data Descriptive

One-minute intervals of SET50 Index options are obtained from Bloomberg, accounted by the Faculty of Economics, Chiang Mai University and Research Institute, Stock Exchange of Thailand. The sample period is from 27 January 2008 until 30 September 2009. The contract months are March, June, September 2008 and 2009 and December 2008.

In order to calculate TVIX, we utilize the SAS 9.1 software package for the calculation as it offers a number of features that are not available in traditional econometric software. For the estimation, we use daily returns of TVIX to estimate ARMA-GARCH, -EGARCH, -GJR-GARCH, and -PGARCH by using E-Views 6.0 software.

The returns of TVIX at time *t* are calculated as follows:

$$\boldsymbol{R_{i,t} = \log(P_{i,t} / P_{i,t-1})} \tag{5.10}$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of TVIX at time *t* and *t-1*, respectively.

Table 5.1: Descriptive Statistics of TVIX Returns

Mean	Std Dev	Skewness	Kurtosis	Max	Min	Jarque-Bera
-0.00022	0.09381	-1.08530	11.41235	0.44728	-0.52266	1204.520*

Note: * significant at the 1% level.

Table 5.1 presents the descriptive statistics for the returns of TVIX. The average return of TVIX is negative. The normal distribution has a skewness statistic equal to zero and a kurtosis statistic of 3, but return of TVIX has negative skewness statistics and high kurtosis, suggesting the presence of fat tails and a non symmetric series. This means that the data has a longer left tail (extreme losses) than right tail (extreme gain). The relatively large kurtosis indicates non-normality that the distribution of returns is leptokurtic. This suggests that the market shocks of either sign for the TVIX returns are more likely to be observed. Jarque-Bera normality test rejects the hypothesis of normality for the sample.

Figure 5.1 presents the plot of TVIX and TVIX returns. This indicates some circumstances where TVIX returns fluctuate. Table 5.2 summarized the unit root tests for TVIX returns. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. The tests yield large negative values in all cases for levels such that the individual returns series reject the null hypothesis at the 1% significance level, hence, the returns are stationary.

Figure 5.1: Daily TVIX and returns

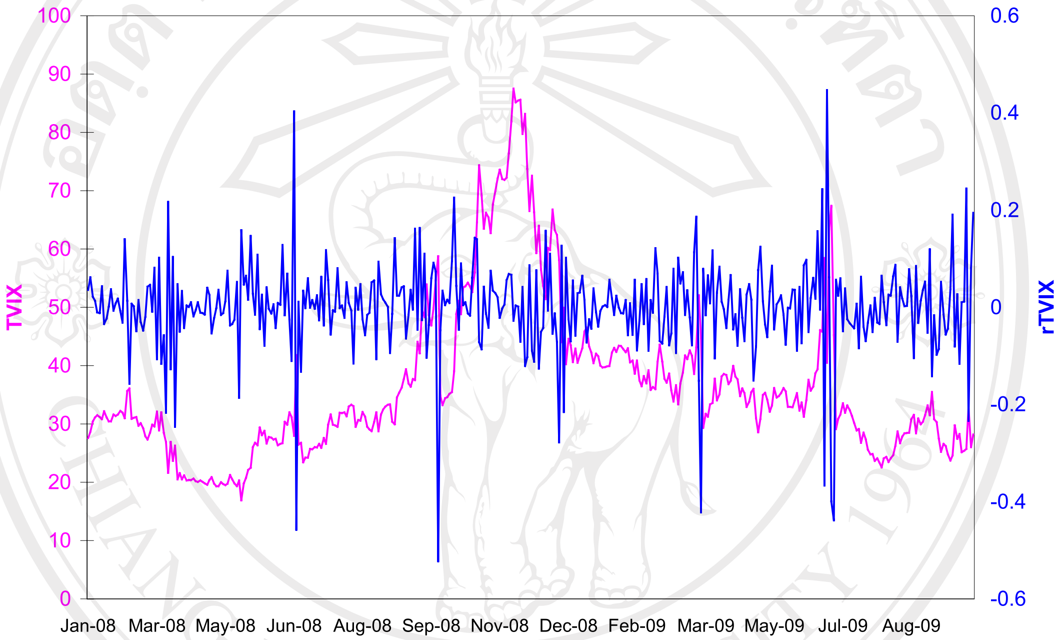

Table 5.2: Unit Root Test for Returns of TVIX

	ADF Test			Phillips-Perron Test		
Returns	**None**	**Constant**	**Constant and Trend**	**None**	**Constant**	**Constant and Trend**
TVIX	-24.022*	-23.991*	-23.977*	-24.560*	-24.526*	-24.522*

Note: * significant at the 1% level.

Table 5.3 represents the ARCH and GARCH effects from statistically significant at 5% level of α and β. It shows that the long-run coefficients are all statistically significant in the variance equation. The coefficients of α appears to show the presence of volatility clustering in the models. Conditional volatility for the models tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). The coefficients of β (a determinant of the degree of persistence) for all models are less than 1 showing persistent volatility.

However, the coefficients of γ, the asymmetry and leverage effects, are negative and statistically significant at the 1% level in the GJR-GARCH and PGARCH models and positive and statistically significant at the 1% level in the EGARCH model. However, the leverage effect only exists if $\gamma < 0$ in the EGARCH model and $\gamma > 0$ in the GJR-GARCH and PGARCH models. This appears that there is asymmetric in all models as $\gamma \neq 0$ but the hypothesis of leverage effect is rejected for all models.

For GARCH(1,1) and GJR(1,1), the results are shown on Table 5.4. The second moment condition is only calculated, and it can be used to verify consistency and asymptotic normality of QMLE in the event that the log-moment condition cannot be computed because $((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1))$ less than zero for any t = 1, 2, …, n (McAleer et al. (2009)). The second moment condition shows the satisfaction rate, the value of which is less than unity in all cases. Hence, the consistency and asymptotic normality of the QMLE are guaranteed.

5.5 Estimation

Table 5.3 represents the ARCH and GARCH effects from statistically significant at 5% level of α and β. It shows that the long-run coefficients are all statistically significant in the variance equation. The coefficients of α appears to show the presence of volatility clustering in the models. Conditional volatility for the models tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). The coefficients of β (a determinant of the degree of persistence) for all models are less than 1 showing persistent volatility.

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In terms of the lowest AIC criteria, the best model is the PGARCH model but in terms of the lowest SBIC, the best model is the EGARCH model. From table 5, the ARMA-GJR has the lowest MAPE and RMSE. In addition, GJR-GARCH model is satisfied by the second moment that is a sufficient condition for the consistency and asymptotic normality of the QMLE. Therefore, GJR-GARCH is the best model.

Table 5.3: Family of GARCH on Returns of TVIX

	Mean Equation			Variance Equation			
GARCH							
	Coefficient	**z-Statistic**		**Coefficient**	**z-Statistic**	**AIC**	**SBIC**
Constant (Mean)	-0.0000	-0.0359 (-0.9714)	ω	0.0004*	5.4432 (0.0000)	-3.899	-3.828
AR(1)	-0.7184**	-8.3029 (0.000)	α	0.7149*	7.7283 (0.0000)		
MA(1)	0.5647**	4.8369 (0.000)	β	0.2441*	3.5503 (0.0000)		
MA(31)	-0.0772**	-2.4124 (0.0158)					
E-GARCH							
	Coefficient	**z-Statistic**		**Coefficient**	**z-Statistic**	**AIC**	**SBIC**
Constant (Mean)	0.0016	1.1975 (0.2311)	ω	0.3676*	-5.2149 (0.0000)	-3.921	-3.839
AR(1)	-0.7025**	-6.9355 (0.000)	α	0.0826*	8.3123 (0.0000)		
MA(1)	0.5432**	4.1779 (0.0000)	β	0.7896*	2.6898 (0.0072)		
MA(31)	-0.0717**	-2.2250 (0.0261)	γ	0.0579*	15.9725 (0.0000)		
GJR-GARCH							
	Coefficient	**z-Statistic**		**Coefficient**	**z-Statistic**	**AIC**	**SBIC**
Constant (Mean)	0.0014	1.0001 (0.3173)	ω	0.0004*	5.5956 (0.0000)	-3.910	-3.828
AR(1)	-0.7149**	-8.1660 (0.0000)	α	1.0561*	4.9896 (0.0000)		
MA(1)	0.5670**	4.7735 (0.0000)	β	0.2525*	3.4095 (0.0007)		
MA(31)	-0.0749**	-2.4047 (0.0162)	γ	-0.6393*	-2.8496 (0.0044)		
PGARCH							
	Coefficient	**z-Statistic**		**Coefficient**	**z-Statistic**	**AIC**	**SBIC**
Constant (Mean)	0.0017	1.4412 (0.1495)	ω	0.0134	1.1377 (0.2553)	-3.925	-3.833
AR(1)	-0.7115*	-8.2320 (0.0000)	α	0.4307*	6.5486 (0.0000)		
MA(1)	0.5603*	4.9346 (0.0000)	β	0.4526*	5.6521 (0.0000)		
MA(31)	-0.0804*	-2.7512 (0.0059)	γ	-0.2951*	-3.5959 (0.0003)		
			δ	0.8738*	3.7207 (0.0002)		

Note: * Significant at the 1% level
** Significant at the 5% level

Table 5.4: Second moment condition for ARMA-GARCH and ARMA-GJR

ARMA-GARCH	**ARMA-GJR**
0.9590	0.9890

Table 5.5: Comparison of test statistics for family of GARCH

	MAPE	**RMSE**
ARMA-GARCH	134.9544	0.093361
ARMA-EGARCH	133.0396	0.093453
ARMA-GJR	132.4593	0.093450
ARMA-PGARCH	134.3143	0.093484

5.6 Conclusion

This paper calculates the Thailand volatility index (TVIX) by applying CBOE Volatility Index (VIX) and SET50 Index options data, and estimates the volatility of TVIX returns using ARMA-GARCH, -EGARCH, -GJR-GARCH, and -PGARCH models. Volatility persistence and asymmetric properties are analyzed.

The results from all of the models show the volatility with statistically significant asymmetry effect with all the models but without leverage effects. This is in contrast to the work of Nelson (1991). The ARMA-PGARCH is found to be the best model with the lowest AIC criteria values but the EGARCH model has the lowest SBIC criteria value. Regarding MAPE and RMSE criteria, GJR-GARCH is the best fitting model for TVIX.