Chapter 2

Methodology

This dissertation constructs indexes of volatility for the USA, Europe, and ASEAN by two ways are single index model and portfolio model, and then compare index of volatility and volatility index by using the predictive power of Value-at-Risk. Moreover, this dissertation finds out volatility spillover from the USA and Europe to ASEAN countries, test change conditional correlation between ASEAN countries and the USA, and between ASEAN countries and Europe following Asian crisis. Furthermore, this dissertation test whether conditional correlation are dynamics by using rolling windows, and examine the impact of the Asian crisis to the VaR thresholds. Therefore, methodologies in this dissertation are index of volatility, Value-at-Risk, and test statistic for testing differences in correlations, rolling windows, and examine the impact of the Asian crisis to the VaR thresholds in section 2.1, 2.2, 2.3, 2.4, and 2.5, respectively.

2.1 Index of Volatility

This dissertation uses the price sector indices of S&P 500 for the USA and STOXX for Europe. There are 10 sector indices; however this dissertation aggregates price sector indices to be 3 sectors by using market capitalization as a weighted variable. For example, if we would like to aggregate sector *1*, *2*, *3* together, the model is as follows:

$$P_{123t} = \frac{MV_{1t} \times P_{1t} + MV_{2t} \times P_{2t} + MV_{3t} \times P_{3t}}{MV_{1t} + MV_{2t} + MV_{3t}} \quad (2.1)$$

where P_{123t} is the aggregate price sector index of sector *1,2,* and *3*, MV_{it} is market capitalization of sector i (i = 1, 2, 3), and P_{it} is price sector index of sector i (i = 1, 2, 3).

Then we compute returns of each sector as follows:

$$R_{i,t} = 100 \times \log(P_{i,t} / P_{i,t-1}) \quad (2.2)$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of sector i (i = 1, 2, 3) at days t and $t-1$.

For ASEAN, we use stock indices of Indonesia, Thailand, and the Philippines to compute returns following equation (2.2). Then we construct an index of volatility for the USA, Europe and ASEAN by using the two following models:

2.1.1 Single index model

This dissertation constructs a single index model following these steps:

(1) Compute portfolio returns by using market capitalization at the first day as a weighted variable, as follows:

$$Port_t = \frac{MV_1 \times r_{1t} + MV_2 \times r_{2t} + MV_3 \times r_{3t}}{MV_1 + MV_2 + MV_3} \quad (2.3)$$

where $Port_t$ is portfolio returns, MV_i is market capitalization of sector i (i = 1, 2, 3), and r_{it} is returns of sector i (i = 1, 2, 3).

For ASEAN, we compute portfolio returns by assumed equal weight in every country.

(2) Test stationary of portfolio returns, this dissertation uses the Augmented Dicky Fuller (ADF) test. The test is given as follows:

$$\Delta y_t = \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.4}$$

$$\Delta y_t = \alpha + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.5}$$

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.6}$$

where equation (2.4) has no intercept and trend, equation (2.5) has intercept but no trend, and equation (2.6) has intercept and trend. The null hypothesis in equations (2.4), (2.5) and (2.6) are $\theta = 0$, which means that y_t is nonstationary (Dickey and Fuller, 1979). However, the ADF test accommodates serial correlation by explicitly modeling the structure of serial correlation, but not heteroscedasticity, while the Phillips-Perron (PP) tests accommodates both serial correlation and heteroscedasticity using non-parametric techniques. The PP test has also been shown to have higher power in finite samples than the ADF test (Phillips and Perron, 1988).

The PP test estimates as follows:

$$\Delta y_t = \theta y_{t-1} + x_t' \delta + \varepsilon_t \tag{2.7}$$

the test is evaluated using a modified t-ratio of the form:

$$\hat{t}_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0}\right)^{1/2} - \frac{T\left(f_0 - \gamma_0\right)\left(se(\hat{\alpha})\right)}{2 f_0^{1/2} s}$$

where $\hat{\alpha}$ is the estimate, t_α is the t-ratio of $\hat{\alpha}$, $se(\hat{\alpha})$ is the standard error of $\hat{\alpha}$, and s is the standard error of the regression. In addition, γ_0 is a consistent estimate of the error variance in (2.7). The remaining f_0 is an estimator of the residual spectrum at frequency zero. The PP test is known as the non-augmented Dickey-Fuller test.

(3) Estimating univariate volatility of portfolio returns from the first step by mean equation have constant term and autoregressive term (AR(1)) in all models. The univariate volatility is the index of volatility. Moreover, this dissertation computes Riskmetrics™ by using the exponentially weighted moving average model (EWMA) of portfolio returns.

Univariate Volatility

ARCH

Engle, R.F. (1982) proposed the Autoregressive Conditional Heteroskedasticity of order p, or ARCH(p), follows:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 \qquad (2.8)$$

where $\omega > 0$ *and* $\alpha_j \geq 0$

GARCH

Bollerslev, T. (1986) generalized ARCH(p) to the GARCH(p,q), model as follows:

$$h_t = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \qquad (2.9)$$

where $\omega > 0$, $\alpha_j \geq 0$ for j = 1,…,p and $\beta_i \geq 0$ for i = 1,…,q are sufficient to ensure that the conditional variance $h_t > 0$.

The model also assumes positive shock ($\varepsilon_t > 0$) and negative shock ($\varepsilon_t < 0$) of equal magnitude have the same impact on the conditional variance.

GJR

Glosten, L.R., et al. (1993) accommodate differential impact on the conditional variance of positive and negative shocks of equal magnitude. The GJR(p,q) model is given by:

$$h_t = \omega + \sum_{j=1}^{p} \left(\alpha_j + \gamma_j I(\varepsilon_{t-j}) \right) \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \qquad (2.10)$$

where the indicator variable, $I(\varepsilon_t)$, is defined as: $I(\varepsilon_t) = \begin{cases} 1, \varepsilon_t \leq 0 \\ 0, \varepsilon_t > 0 \end{cases}$. If p = q = 1, $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \gamma_1 \geq 0$, and $\beta_1 \geq 0$ then it has sufficient conditions to ensure that the conditional variance h_t > 0. The short run persistence of positive (negative) shocks is given by $\alpha_1 (\alpha_1 + \gamma_1)$. When the conditional shocks, η_t, follow a symmetric distribution, the short run persistence is $\alpha_1 + \gamma_1 / 2$, and the contribution of shocks to long run persistence is $\alpha_1 + \gamma_1 / 2 + \beta_1$.

EGARCH

Nelson, D. (1991) proposed the Exponential GARCH (EGARCH) model, which incorporates asymmetries between positive and negative shocks on conditional volatility. The EGARCH model is given by:

$$\log h_t = \omega + \sum_{i=1}^{p} \alpha_i \left|\eta_{t-i}\right| + \sum_{i=1}^{p} \gamma_i \eta_{t-i} \sum_{j=1}^{q} \beta_j \log h_{t-j} \qquad (2.11)$$

In equation (2.11), $\left|\eta_{t-i}\right|$ and η_{t-i} capture the size and sign effects, respectively, of the standardized shocks. EGARCH in (2.11) uses the standardized residuals. As EGARCH uses the logarithm of conditional volatility, there are no restrictions on the parameters in (2.11). As the standardized shocks have finite moments, the moment conditions of (2.11) are straightforward.

Lee, S.W. and Hansen, B.E. (1994) derived the log-moment condition for GARCH (1,1) as

$$E(\log(\alpha_1 \eta_t^2 + \beta_1)) < 0 \qquad (2.12)$$

This is important in deriving the statistical properties of the QMLE. McAleer, M., et al. (2007) established the log-moment condition for GJR(1,1) as

$$E(\log((\alpha_1 + \gamma_1 I(\eta_t))\eta_t^2 + \beta_1)) < 0 \tag{2.13}$$

The respective log-moment conditions can be satisfied even when $\alpha_1 + \beta_1 > 1$(that is, in the absence of second moments of the unconditional shocks of the GARCH(1,1) model) and when $\alpha_1 + \gamma / 2 + \beta_1 < 1$(that is, in the absence of second moments of the unconditional shocks of the GJR(1,1) model).

Riskmetrics™

Riskmetrics™ (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is, in effect, a restricted version of the ARCH(∞) model. This approach forecasts the conditional variance at time t as a linear combination of lagged conditional variance and the squared unconditional shock at time *t-1*. The Riskmetrics™ model estimate the conditional variances follows:

$$h_t = \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \tag{2.14}$$

where λ is a decay parameter. Riskmetrics™ (1996) suggests that λ should be set at 0.94 for purposes of analyzing daily data.

2.1.2 Portfolio model

This dissertation constructs the portfolio model by following these steps:

(1) Test stationary of stock returns, this dissertation uses the Augmented Dicky Fuller (ADF) test. The test is given as follows:

$$\Delta y_t = \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.15}$$

$$\Delta y_t = \alpha + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.16}$$

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \tag{2.17}$$

where equation (2.15) has no intercept and trend, equation (2.16) has intercept but no trend, and equation (2.17) has intercept and trend. The null hypothesis in equations (2.15), (2.16) and (2.17) are $\theta = 0$, which means that y_t is nonstationary (Dickey and Fuller, 1979). However, the ADF test accommodates serial correlation by explicitly modeling the structure of serial correlation, but not heteroscedasticity, while the Phillips-Perron (PP) tests accommodates both serial correlation and heteroscedasticity using non-parametric techniques. The PP test has also been shown to have higher power in finite samples than the ADF test (Phillips and Perron, 1988).

The PP test estimates as follows:

$$\Delta y_t = \theta y_{t-1} + x_t' \delta + \varepsilon_t \tag{2.18}$$

the test is evaluated using a modified t-ratio of the form:

$$\hat{t}_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0} \right)^{1/2} - \frac{T\left(f_0 - \gamma_0\right)\left(se(\hat{\alpha})\right)}{2 f_0^{1/2} s}$$

where $\hat{\alpha}$ is the estimate, t_α is the t-ratio of $\hat{\alpha}$, $se(\hat{\alpha})$ is the standard error of $\hat{\alpha}$, and s is the standard error of the regression. In addition, γ_0 is a consistent estimate of the error variance in (2.18). The remaining f_0 is an estimator of the residual spectrum at frequency zero. The PP test is known as the non-augmented Dickey-Fuller test.

(2) Estimate multivariate volatility of three sectors for Europe and the USA and three countries for ASEAN by mean equation so that they have constant term and autoregressive term (AR(1)) in all models. Then compute variance and covariance matrix.

(3) Compute index of volatility by using market capitalization at the first observation is a weighted variable. This dissertation has three sectors so that we have the three conditional variances and three covariance estimated. It follows that:

$$IVol_t = \lambda_1^2 h_{1t} + \lambda_2^2 h_{2t} + \lambda_3^2 h_{3t} + 2\lambda_1\lambda_2 h_{12t} + 2\lambda_1\lambda_3 h_{13t} + 2\lambda_2\lambda_3 h_{23t} \quad (2.19)$$

where $IVol_t$ is index of volatility, h_{it} is conditional variances of sector i (i=1,2,3), h_{ijt} is covariance of sector i (i=1,2,3), and $\lambda_1 = \frac{MV_1}{MV_1 + MV_2 + MV_3}$,

$\lambda_2 = \frac{MV_2}{MV_1 + MV_2 + MV_3}$, and $\lambda_3 = \frac{MV_3}{MV_1 + MV_2 + MV_3}$.

The number of covariance increases dramatically with m, the number of assets in the portfolio. Thus, for m = 2, 3, 4, 5, 10, 20, the number of covariance is 1, 3, 6, 10, 45, 190, respectively. This increases the computation burden significantly. (See details in McAleer, M., 2008)

For ASEAN, we assumed equal portfolio weight for all assets. Therefore, λ_1, λ_2, and λ_3 in equation (2.19) are equal 1/3.

Multivariate volatility

VARMA-GARCH

The VARMA-GARCH model of Ling, S. and McAleer, M. (2003), assumes symmetry in the effects of positive and negative shocks of equal magnitude on conditional volatility. Let the vector of returns on $m\ (\geq 2)$ financial assets be given by:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t \tag{2.20}$$

$$\varepsilon_t = D_t \eta_t \tag{2.21}$$

$$H_t = \omega + \sum_{k=1}^{p} A_k \vec{\varepsilon}_{t-k} + \sum_{l=1}^{q} B_l H_{t-l} \tag{2.22}$$

where $H_t = (h_{1t},...,h_{mt})'$, $\omega = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{i,t}^{1/2})$, $\eta_t = (\eta_{1t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)'$, A_k and B_l are $m \times m$ matrices with typical elements α_{ij} and β_{ij}, respectively, for i,j = 1,…,m, and F_t is the past information available to time t. Spillover effects are given in the conditional volatility for each asset in the portfolio, specifically where A_k and B_l are not diagonal matrices. For the VARMA-GARCH model, the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$.

VARMA-AGARCH

An extension of the VARMA-GARCH model is the VARMA-AGARCH model of McAleer, M., et al. (2009), which assumes asymmetric impacts of positive and negative shocks of equal magnitude, and is given by:

$$H_t = \omega + \sum_{k=1}^{p} A_k \vec{\varepsilon}_{t-k} + \sum_{k=1}^{p} C_k I_{t-k} \vec{\varepsilon}_{t-k} + \sum_{l=1}^{q} B_l H_{t-l} \quad (2.23)$$

where C_k are $m \times m$ matrices for k = 1,…,p and I(η_t)=diag(I(η_{it})) is an $m \times m$ matrix, so that $I = \begin{cases} 0, \varepsilon_{k,t} > 0 \\ 1, \varepsilon_{k,t} \le 0 \end{cases}$. VARMA-AGARCH reduces to VARMA-GARCH when C_k =0 for all k.

CCC

If the model given by equation (2.23) is restricted so that C_k = 0 for all k, with A_k and B_l being diagonal matrices for all k,l, then VARMA-AGARCH reduces to:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_i \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_i h_{i,t-l} \quad (2.24)$$

Which is the constant conditional correlation (CCC) model of Bolerslev, T. (1990), for which the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$. As given in equation (2.24), the CCC model does not have volatility spillover effects across different financial assets, and does not allow conditional correlation coefficients of the returns to vary over time.

DCC

Engle, R.F. (2002) proposed the Dynamic Conditional Correlation (DCC) model. The DCC model can be written as follows:

$$y_t \mid F_{t-1} \sim (0, Q_t), \quad t = 1, ..., T \quad (2.25)$$

$$Q_t = D_t \Gamma_t D_t, \tag{2.26}$$

where $D_t = diag(h_{1t}^{1/2},...,h_{mt}^{1/2})$ is a diagonal matrix of conditional variances, with m asset returns, and F_t is the information set available at time t. The conditional variance is assumed to follow a univariate GARCH model, as follows:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_{i,k} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{i,l} h_{i,t-l} \tag{2.27}$$

When the univarate volatility models have been estimated, the standardized residuals, $\eta_{it} = y_{it} / \sqrt{h_{it}}$, are used to estimate the dynamic conditional correlations, as follows:

$$Q_t = (1-\phi_1-\phi_2)S + \phi_1 \eta_{t-1} \eta'_{t-1} + \phi_2 Q_{t-1} \tag{2.28}$$

$$\Gamma_t = \left\{(diag(Q_t)^{-1/2}\right\} Q_t \left\{(diag(Q_t)^{-1/2}\right\}, \tag{2.29}$$

where S is the unconditional correlation matrix of the returns shocks, and equation (2.29) is used to standardize the matrix estimated in (2.28) to satisfy the definition of a correlation matrix. For details regarding the regularity conditional and statistical properties of DCC and the more general GARCC model, see McAleer, M., et at. (2008).

The parameters in models (2.8), (2.9), (2.10), (2.11), (2.22), (2.23), (2.24), and (2.27) can be obtained by maximum likelihood estimation (MLE) using a joint normal density, as follows:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{2} \sum_{t=1}^{n} (\log|Q_t| + \varepsilon_t' Q_t^{-1} \varepsilon_t) \quad (2.30)$$

where θ denotes the vector of parameters to be estimated in the conditional log-likelihood function, and $|Q_t|$ denotes the determinant of Q_t, the conditional covariance matrix. When η_t does not follow a joint normal distribution, equation (2.30) is defined as the Quasi-MLE (QMLE).

Then, compare index of volatility and volatility index by using the predictive power of Value-at-Risk. The details of Value-at-Risk are in section 2.2.

2.2 Value-at-Risk

Value-at-Risk (VaR) needs to be provided to the appropriate regulatory authority at the beginning of the day, and is then compared with the actual returns at the end of the day. (see McAleer, M., 2008)

For the purposes of the Basel II Accord penalty structure for violations arising from excessive risk taking, a violation is penalized according to its cumulative frequency of occurrence in 250 working days, which is shown in Table 2.1.

A violation occurs when VaR_t > negative returns at time t. Suppose that interest lies in modeling the random variable Y_t, which can be decomposed as follows (see McAleer, M. and da Veiga, B., 2008a):

$$Y_t = E(Y_t | F_{t-1}) + \varepsilon_t \quad (2.31)$$

This decomposition suggests that Y_t is comprised of a predictable component, $E(Y_t \mid F_{t-1})$, which is the conditional mean, and a random component, ε_t. The variability of Y_t, and hence its distribution, is determined entirely by the variability of ε_t. If it is assumed that ε_t follows a distribution such that:

$$\varepsilon_t \sim D(\mu_t, \sigma_t) \tag{2.32}$$

where μ_t and σ_t are the unconditional mean and standard deviation of ε_t, respectively, the VaR threshold for Y_t can be calculated as:

$$VaR_t = E(Y_t \mid F_{t-1}) - \alpha\sigma_t$$

where α is the critical value from the distribution of ε_t to obtain the appropriate confidence level. Alternatively, σ_t can be replaced by alternative estimates of the conditional variance to obtain an appropriate VaR.

The Basel II encourages the optimization problem with the number of violations and forecasts of risk as endogenous choice variables, which are as follows:

$$\underset{\{k, VaR_t\}}{Minimize} \quad DCC_t = \max\left\{-(3+k)\overline{VaR}_{60}, -VaR_{t-1}\right\} \tag{2.33}$$

where DCC is daily capital charges, k is a violation penalty ($0 \le k \le 1$) (see Table 2.1), $\overline{VaR}_{60}$ is mean VaR over the previous 60 working days, and VAR_t is Value-at-Risk for day *t*.

2.3 Test statistic for testing differences in correlations

This dissertation also would like to find out about volatility spillover from Europe and the USA to ASEAN countries. Countries in ASEAN which use in this section are names, Indonesia, Malaysia, the Philippines, Singapore, and Thailand. We calculate returns follow equation (2.2) of stock price indices of Indonesia, Malaysia, the Philippines, Singapore, Thailand, the USA and Europe and we use VARMA-AGARCH model follow equation (2.23) to find out returns and volatility spillover from the USA and Europe to ASEAN countries by mean equation have constant term, autoregressive (AR(1)) term, and moving average (MA(1)) term. We also would like to test whether the Asian crisis affect conditional correlation between ASEAN countries and the USA, Europe. Therefore, we estimate VARMA-AGARCH model follow equation (2.23) for the entire sample (5 January 1988 to 13 March 2009), the sub-sample before the Asian crisis (5 January 1988 to 27 December 1996) and the sub-sample after the crisis (5 January 1998 to 13 March 2009) to find out conditional correlation matrices between ASEAN countries, Europe and the USA. Let ρ_1 and ρ_2 be the correlations from the after and before Asian crisis period, respectively. The test statistic for testing differences in correlations is then given by

$$Z = \frac{\rho_1 - \rho_2}{S.E.} \tag{2.34}$$

$$S.E. = \sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}} \tag{2.35}$$

where n_1 and n_2 are sample sizes used to calculate ρ_1 and ρ_2, respectively.

2.4 Rolling windows

Using the 'rolling windows' approach, we can examine the time-varying nature of the conditional correlation using the VARMA-AGARCH model. Rolling windows is a recursive estimation procedure whereby the model is estimated for a restricted sample, then re-estimated by adding one observation to the end of the sample and deleting one observation from the beginning of the sample. The process is then repeated until the end of the sample. If the rolling conditional correlations are found to vary substantially over time, the assumption of constant conditional correlations may be too restrictive. In order to strike a balance between efficiency in estimation, and a viable number of rolling regressions, the rolling window size is set at 1,000.

2.5 Examine the impact of the Asian crisis to the VaR thresholds

This section constructs portfolio returns of each country in ASEAN with Europe and the USA, and in order to eliminate exchange rate risk, all returns are converted to US dollars. Then forecast the VaR thresholds for the period 3 January 2007 to 13 March 2009 by using observation from the previous year, 2006, and the number of violations is recorded. The sample is then expanded by adding observations from next previous year, 2005, to the beginning of the sample (1988), and again the VaR threshold for the period 3 January 2007 to 13 March 2009 is forecasted. This process is repeated until the beginning of the sample is reached. For the details of the VaR threshold are in section 2.2

Table 2.1 Basel Accord Penalty Zones

Zone	Number of Violations	Increase in k
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days.